

The Competition Between Thermal Contraction and Differentiation in the Stress History of the Moon

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The scarcity of both extension and compression features on the Moon strongly constrains the history of the lunar radius—to variations of less than ± 1 km over the past 3.8 Gyr. This limit has traditionally been interpreted as requiring a delicate balance between thermal contraction of the near-surface and expansion of a substantial cold interior region. Recent theories of lunar origin (e.g., giant impact), in contrast, favor a “hot” initial state. We propose that a reconciliation may be possible by taking account of the volume change $\Delta V/V|_d$ due to differentiation. We calculate STP densities based on simplified normative mineralogies for a suite of estimates of the bulk lunar composition, of primary lunar basalt, and of the residuum left when the maximum amount of the latter is extracted from the former. Typically $\Delta V/V|_d \approx 2$ to 5%—an expansion equivalent to heating by $\sim 10^3$ K. Provided the timing of differentiation is correct, one might offset the cooling of a magma ocean as much as 630 km deep by differentiation of the remainder of the Moon (which need not start much below the solidus temperature). A large but not impossible amount of gabbroic melt production is implied: ~ 100 times the volume of mare basalts known to have been extruded. We do not address the detailed genetic relationship of this melt to the basalts observed on the lunar surface but point out that it need not have reached the surface directly or even have entered the crust in order for the expansion to have occurred. To assess the timing of melt formation, we investigate a simple conductive lunar thermal model which takes account of both $\Delta V/V|_d$ and thermal contraction. Our initial state is characterized by a central temperature T_c and a depth Z_0 above which the material (derived from the magma ocean) is already at the solidus and is not susceptible to volume changes upon further differentiation. We find a range of models satisfying the limits on radius increase and decrease. The hottest has $T_c = 1210$ K, $Z_0 = 400$ km; without $\Delta V/V|_d$, we would need a larger or colder (or both) core, e.g., $T_c \lesssim 700$ K for $Z_0 = 200$ –400 km, in agreement with previous investigators. Our modeling thus lends credence to the idea that the Moon could have been initially $\gtrsim 50\%$ molten (with the remainder relatively close to the solidus) and yet experienced little volume change over the last 3.8 Gyr.

INTRODUCTION

Volumetric changes throughout the interior of an evolving solid planet will inevitably be felt at the surface as stresses, potentially making themselves manifest to the terrestrial observer through the creation of extensional or compressional tectonic features. Thus the surface of Mercury betrays an epoch of internal contraction, while that of Mars is indicative of substantial global expansion. The case of the Moon, on the other hand, is reminiscent of “the curious incident of the dog in the night-time” famous in *Holmesiana*: the dog did nothing in the night-time. Solomon and Chaikin [1976] demonstrated that the absence of tectonic features expected if the lunar radius had varied by $\gtrsim 1$ km [MacDonald, 1960] provides a strong constraint against which to test models of the Moon’s thermal history. The limits on radius variation per se apply only to the past 3.8 Gyr, since tectonic features from an earlier era may have been obliterated by the tail end of heavy impact bombardment. One may nonetheless use them as a discriminant of theories of the formation of the Moon, to the extent that one is confident of knowing the physical processes by which that body evolves. Knowing the differential equation, one seeks the initial condition.

Solomon and Chaikin [1976] computed the thermal volume change $\int \int \alpha \partial T / \partial t dV dt$ for a suite of conductive thermal models and found that if the radius constraint is to be satisfied, the contraction of the outermost layers of the Moon must be balanced by the radiogenic warming and expansion of a substantial cold core. (Unless the contrary is directly indicated, we use the term “core” in this paper to denote the initially solid inner regions of the Moon of whatever composition, rather than the metallic iron region which is commonly meant.) Subsequent work [Solomon, 1977; Solomon and Head, 1979] refined and substantiated this conclusion. A cold core comprising 60–80% of the volume of the Moon, with a central temperature of no more than ~ 700 K, would seem to be required. These are distressing conclusions, in light of the current interest in models of lunar formation which lead to total or near-total melting.

Extensive melting of the Moon just after accretion because of short-lived radionuclides has been proposed by Runyon [1977]. Melting is also an expected concomitant of formation of the Moon by binary fission [O’Keefe, 1969; Wise, 1969; Binder, 1980]. The strongest stimulus to renewed interest in a hot early Moon, however, has been a class of theories of lunar origin involving the impact of a Mars-sized body on the Earth [Hartmann and Davis, 1975; Cameron and Ward, 1976]. Both modeling of a circumterrestrial accretion disk formed by such an impact [Thompson and Stevenson, 1983; Cameron, 1984; Stevenson, 1984, 1987; A. C. Thompson and D. J. Stevenson, Gravitational instability of two-phase disks and the origin of the Moon, submitted to *Astrophysical Journal*, 1987] and numerical simulation of the impact process itself [Benz et al., 1986,

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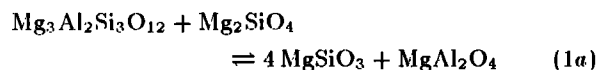
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1987] lead to the expectation that the source material for the Moon underwent complete melting. Because of the success (or at least lack of demonstrated failure) of the giant impact models in explaining other properties of the Moon, clarification of their consequences for tectonic activity is of the greatest interest.

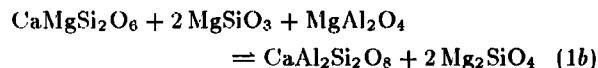
Binder and Lange [1980] and Binder [1982, 1986] have claimed that the tectonic evidence on the Moon is consistent with extensive early melting. In addition to constructing initially hot thermal evolution models which undergo less contraction than those of Solomon and Chaikin [1976] (but still $\gg 1$ km), they argued for a substantially stronger lunar lithosphere. At the same time, they argued that compressional failure may actually be observed on the Moon both in the form of high-frequency teleseisms (shallow moonquakes) and as the cause of highland scarps, and that stress may have been further relieved by undetectable slip on faults of the "lunar grid." These claims have been challenged [Solomon, 1986], and at best their validity is far from clear. The problem of the tectonic history of an initially molten Moon cannot be considered solved.

Our purpose in this paper is to point out that existing attempts to apply the volumetric constraint to the Moon's early thermal history have been inadequate in the physics they contain and hence possibly misleading. (Not knowing the proper differential equation, one arrives at the wrong initial condition.) Thermal expansion is not the sole possible contributor to evolutionary changes in the lunar radius. The net change in volume accompanying melting, magma migration, and refreezing, which has been neglected even in those studies which include melting in the energy balance, is potentially as important.

This differentiation volume change is well known in the case of the Earth. Oxburgh and Parmentier [1977] calculated the densities of a model upper mantle, basaltic crust, and the depleted region produced by extracting the latter from the former. Both basalt and depleted mantle were shown to be buoyant with respect to the undepleted mantle at the same temperature. The authors assumed a mass yield of basalt from undepleted mantle of 25%, and obtained densities and layer thicknesses from which a net expansion of the differentiated region of 4.55% may be calculated. The primary cause of this expansion is clear: upward migration of the melt carries Al_2O_3 from the region of stability of garnet to that of plagioclase. The transformation proceeds in two stages. These are (for the magnesium end-member species):



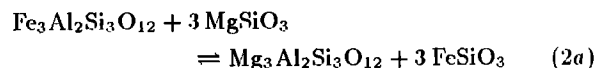
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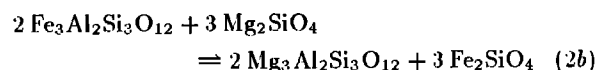
The fractional change in volume at standard temperature and pressure for the first reaction is an extraordinary 26.6%, and for the second, 1.4%. The net transformation of garnet to plagioclase is accompanied by a 24.5% expansion (mineral densities are taken from *Basaltic Volcanism Study Project* [BVSP] [1981]). Even when account is taken of the lesser fraction of the whole rock which participates in the first reaction, it accounts for roughly nine-tenths of the total

volume change $\Delta V/V|_d$ in the lunar case. The pressure-temperature-composition phase boundaries for these reactions have been investigated using both natural mineral mixtures and idealized synthetic systems [Green and Ringwood, 1967; MacGregor, 1964, 1965, 1970]. At temperatures appropriate for early lunar history (cf. Figure 5), the plagioclase-spinel transition occurs at a depth of 180–200 km in the Moon ($P \simeq 1$ GPa) and the spinel-garnet transition at 400–500 km (2.0–2.3 GPa) [BVSP, 1981].

Oxburgh and Parmentier [1977] also cited the preferential extraction of iron from the depleted zone as a cause of the decrease in that region's density. Examination of the exchange reactions



and



which have volume changes of -8.7% and 1.2% , respectively, shows that migration of iron from the garnet to the plagioclase stability field probably results in a small net contraction when the two layers are considered together. We are of course neglecting the effects of iron-magnesium ratio on compressibility, in which approximation the movement of fayalite and ferrosilite from interior to surface does not change the volume. In any case, the effect of Al_2O_3 migration is clearly the dominant one.

This mechanism of expansion will operate in the Moon as well as the Earth, provided differentiation transports susceptible material upward out of the garnet stability field. It is also necessary that the downward flow (burial) of country rock required by conservation of mass does not recreate the lost garnet. (For the Earth, the former consideration was of interest to Oxburgh and Parmentier [1977], but the latter was not. The differentiated material is recycled into the mantle by subduction at the same rate at which it is created, and the Earth does not expand with time by this mechanism.) On the other hand, we emphasize that it is not necessary in order that there be a net expansion that the igneous product reach the surface of the body or that it go to make up the conventionally defined crust. There is no direct connection implied between the primary melt responsible for the expansion and the basalts encountered at the lunar surface, with their various and complex fractionation histories. Once the aluminous material has reached the plagioclase (or spinel) stability field, it may be reworked ad libitum without appreciable additional change in volume.

We argue that the conditions for differentiation-driven expansion are quite readily met. As the volume changes for the reactions (1a) and (1b) show, most of the expansion occurs at the garnet-spinel transition, at 400–500 km depth. Basaltic melting below this depth is plausible [Ringwood, 1979] (see also our models in section 4). The majority of the melt will almost certainly ascend to above this point, which is much deeper than typically advocated skin depths even for the earliest stages of differentiation [Toksöz and Solomon, 1973]. As for the return flow, this is likely to be composed primarily of the Al_2O_3 -depleted material formed from the early magma ocean after the fractionation of the

crust. We calculate below that of the order of 0.1 lunar volume of melt will be produced, equivalent to a layer ~ 65 km thick in the outer regions of the Moon. There is thus ample room for both a 70-km anorthosite crust (derived from the magma ocean) and a subsequently formed gabbro layer within the plagioclase stability field. Of course, the intrusive material is likely to solidify over a range of depths rather than in a well-defined layer. We nonetheless conclude, first, that previously formed crustal plagioclase is not likely to be converted to spinel by burial and, second, that some of the gabbro may crystallize in the spinel field but its expansion will be nearly as great as if it had reached the plagioclase field. Aluminous material is unlikely to enter the garnet field in any quantity through burial. The volume effect of differentiation may thus be approximated by the conversion of garnetiferous primordial mantle to residue plus plagioclase-containing basalt.

The volume change for this process may be expected to differ from the corresponding expansion for terrestrial materials, because of compositional differences between the Earth and Moon. In section 2 of this paper we attempt to quantify the amount of expansion caused by extracting basalt from undifferentiated lunar material, taking account of the uncertainty in our knowledge of the appropriate compositions. The resulting estimate of $\Delta V/V|_d$ is used in subsequent sections to compare the relative importance of thermal and differentiation effects in the Moon's volumetric history. In section 3 we make some simple calculations, not tied to any particular thermal evolution models, which show that differentiation is likely to be of major importance. section 4 presents a conductive thermal model and demonstrates the results of including the differentiation volume change. Our discussion, section 5, focuses on the possible relationship between our results and theories of lunar origin.

CALCULATION OF DENSITIES

Interpretations of the composition of the lunar interior are fraught with a number of uncertainties, including but scarcely limited to the current thermal state (whose calculation we seek here to advance). We therefore take an inclusive, rather than exclusive, approach to the estimation of $\Delta V/V|_d$. We take from the literature six widely differing bulk lunar composition models and two suggested models for the primary basaltic melt, and calculate the volume change on differentiation for all combinations. The range of results gives both estimates bounding $\Delta V/V|_d$ within perhaps a factor of 2 and some understanding of the variables most affecting that quantity.

The sources of our compositional models are summarized in Table 1. Of the six bulk Moon models, numbered in order of decreasing Al_2O_3 and CaO content, two are from BVSP [1981], section 4.5.2, and are "adjusted" versions of models by other authors. That is, they have been derived from previously published models by varying a few parameters (magnesium-iron ratio, core size, etc.) to improve the agreement between observed and model values of geophysical quantities such as bulk density, moment of inertia, and seismic profiles. Three more models are among those considered in Hood and Jones [1987], while the sixth is from Buck and Toksöz [1980]. Two potential models from the same sources were not used: BVSP [1981] model Mo3, which could not be adjusted satisfactorily, and Hood and Jones [1987] model 1, which cannot be projected onto the simpli-

TABLE 1. Sources of Compositional Models

Model	Source
M1	BVSP [1981] model Mo1, adjusted from Morgan <i>et al.</i> [1978]
M2	Hood and Jones [1987] model 1, from Taylor [1982]
M3	BVSP [1981] model Mo2, adjusted from Wänke <i>et al.</i> [1977]
M4	Buck and Toksöz [1980]
M5	Hood and Jones [1987] model 3, from Wänke <i>et al.</i> [1977]
M6	Hood and Jones [1987] model 4, from Delano [1984]
B1	Apollo 12 olivine basalt 12009 plus 10% Fo ₇₅ [Green <i>et al.</i> , 1971]
B2	Apollo 15 green glass [Green and Ringwood, 1973]

fied mineralogical scheme we use (see below). The models from BVSP [1981] and Buck and Toksöz [1980] have the advantage of having been fit to selenophysical constraints and seismic velocity estimates. The seismic profiles used [Goins, 1978] differ substantially, however, from those of Nakamura [1983] based on a more complete moonquake dataset. Hood and Jones [1987] tested their models (without adjustment) against the latter under a variety of assumptions about the thermal and differentiation state of the Moon. We will discuss their conclusions below.

Our basalt models are the compositions of two lunar samples thought to be candidates for a primary basaltic melt: the Apollo 12 olivine basalt 12009 modified by the addition of 10% olivine of composition Fo₇₅ [Green *et al.*, 1971], and the Apollo 15 emerald green glass [Green and Ringwood, 1973]. We also carried out calculations for a number of Apollo 15 low-titanium basalts but do not tabulate the results, which were generally similar to those for models B1 and B2.

Recognizing the unavoidable uncertainties due to the range of composition models available, we simplified as far as possible the method by which the volume effect of differentiation was estimated. For each combination of bulk Moon and basalt models, the maximum mass fraction of basalt extractable from the primordial material was calculated. The densities at standard temperature and pressure of the primordial mantle, the basalt, and the residuum after complete melt extraction were then compared to give an estimate of $\Delta V/V|_d$. The neglect of compressibility effects implicit in this method is justified by the relatively low pressures in the lunar interior as well as by our desire for a simple, robust result.

More important, the yields and STP densities were themselves calculated using a simplified mineralogical scheme. We chose to deal with only the oxides SiO_2 , Al_2O_3 , MgO , FeO , and CaO . Minor elements being more susceptible to secondary processes of enrichment and depletion, their inclusion in the calculation of primary basalt yield X_{max} could be misleading. From the published analyses we therefore removed TiO_2 (as ilmenite), Cr_2O_3 (as chromite), MnO (as rhodonite), Na_2O (as jadeite at high pressure, as albite at low pressure), and K_2O (as orthoclase) and renormalized the remaining oxides to 100%. The resulting simplified analyses (in percent by mass) appear in Table 2; for the original analyses the reader is directed to the literature. Removing

TABLE 2. Compositions and Densities of Moon, Basalt, and Residuum Models

	M1	M2	M3	M4	M5
SiO ₂	49.351	44.207	48.096	49.020	44.830
Al ₂ O ₃	7.515	6.023	4.895	4.846	4.445
MgO	25.860	34.732	30.229	29.588	34.429
FeO	11.038	10.406	12.647	12.641	12.536
CaO	6.235	4.632	4.132	3.906	3.761
Σ	99.999	100.000	99.999	100.001	100.001
An					
Ol	6.523	45.909	23.466	17.309	45.828
Opx	37.197	10.774	39.686	44.399	20.778
Di	24.423	18.116	16.182	18.626	14.728
Ga	31.857	25.171	20.667	19.666	18.666
Σ	100.000	100.000	100.001	100.000	100.000
Mg'	0.7954	0.8470	0.7986	0.7952	0.8200
ρ_{STP} , kg m ⁻³	3468.5	3413.9	3437.1	3434.4	3445.3
	M6	B1	B2	M1 - B1	M1 - B2
SiO ₂	41.315	46.796	41.828	52.560	52.174
Al ₂ O ₃	3.519	8.106	7.838	6.774	7.315
MgO	36.286	15.899	17.321	38.372	31.191
FeO	16.065	19.826	21.300		4.632
CaO	2.815	9.373	8.713	2.294	
Σ	100.000	100.000	100.000	100.000	100.000
An		22.058	21.387		
Ol	67.587	19.360	31.789	7.665	
Opx	6.531	39.398	29.589	55.973	51.525
Di	11.023	19.185	17.235	8.983	18.359
Ga	14.859			27.379	30.116
Σ	100.000	100.001	100.000	100.000	100.000
Mg'	0.7894	0.5709	0.5743	1.0000	0.9178
ρ_{STP} , kg m ⁻³	3484.4	3338.8	3365.8	3293.6	3370.6
	M2 - B1	M2 - B2	M3 - B1	M3 - B2	M4 - B1
SiO ₂	41.677	43.613	49.121	51.041	50.609
Al ₂ O ₃	3.988	4.290	2.364	2.241	2.518
MgO	53.134	51.362	41.528	41.873	39.366
FeO	1.201		6.988	4.842	7.508
CaO		0.735			
Σ	100.000	100.000	100.001	100.000	100.001
An					
Ol	81.116	69.432	36.275	27.799	26.133
Opx	2.720	10.350	53.995	63.025	63.480
Di		2.876			
Ga	16.164	17.342	9.731	9.176	10.387
Σ	100.000	100.000	100.001	100.000	100.000
Mg'	0.9866	1.0000	0.9079	0.9348	0.8969
ρ_{STP} , kg m ⁻³	3287.9	3282.3	3333.2	3294.8	3328.5
	M4 - B2	M5 - B1	M5 - B2	M6 - B1	M6 - B2
SiO ₂	52.425	43.512	44.834	38.964	39.639
Al ₂ O ₃	2.415	1.992	1.868	1.550	1.458
MgO	39.554	46.847	47.417	45.036	45.335
FeO	5.605	7.650	5.880	14.450	13.567
CaO					
Σ	99.999	100.001	99.999	100.000	99.999
An					
Ol	17.977	70.459	61.889	92.423	89.529
Opx	72.105	21.346	27.452	1.109	4.396
Di					
Ga	10.387	8.195	7.660	6.468	6.075
Σ	100.000	100.000	100.001	100.000	100.000
Mg'	0.9213	0.9104	0.9305	0.8379	0.8472
ρ_{STP} , kg m ⁻³	3301.0	3337.6	3311.6	3428.6	3414.0

All compositions in weight percent. An, anorthite; Ol, olivine; Opx, orthopyroxene; Di, diopside; Ga, pyrope-almandine garnet; Σ, total. Mg' ≡ MgO/(MgO + FeO) by mole.

the minor minerals, which are largely "inert" (i.e., which refreeze in their original phases) during the differentiation process, from the composition will result in a small overestimate of $\Delta V/V_d$. Since they comprise only about 1–3% of the mantle, this effect is much smaller than the variability between different compositional models.

We further simplify our analysis by neglecting the solid solution of Al in pyroxene, by considering pyrope-almandite garnets but not grossular, and by assuming [Oxburgh and Parmentier, 1977] that the magnesium-iron ratio is the same in olivine, orthopyroxene, and garnet but that no iron is incorporated in clinopyroxene. Our chemical analyses can then be interpreted uniquely in terms of the mineral assemblages olivine-orthopyroxene-diopside-garnet at high pressure and anorthite-olivine-orthopyroxene-diopside at low pressure (Table 2). We do not consider the spinel stability field. As indicated in the introduction, the conversion of garnet to spinel results in much greater expansion than the conversion of spinel to plagioclase. The calculated value of $\Delta V/V_d$ would thus be approximately correct even if the melt were to freeze in the spinel field. Furthermore, this region is occupied in the cases of most interest to us by alumina-poor material solidified from the primordial magma ocean, which would undergo little volume change upon further differentiation.

The equilibria for formation of the neglected aluminous phases are pressure and temperature dependent and somewhat uncertain [Akella, 1976; Perkins *et al.*, 1981]. Their incorporation into our rough, global estimate of $\Delta V/V_d$ would thus be difficult. At any rate, aluminous pyroxenes may comprise 4–6% of the lunar mantle [BVSP, 1981]. Their reaction with orthopyroxene in a 2:1 ratio to form garnet (effectively forced by our simplified scheme) results in a contraction amounting to $\sim 7\%$ of the volume of the reactants. The net effect of neglecting them is thus of the order of 1% in the calculation of SFP densities [cf. Buck and Toksöz, 1980]. Because we neglect the aluminous pyroxenes both before and after differentiation, partial cancellation will cause the effect on $\Delta V/V_d$ to be smaller. The effect on SFP density of converting grossular to pyrope-almandite, evaluated in a similar way, is only $\sim 0.05\%$.

We note in passing, however, that our simplified analysis results in a negative normative orthopyroxene content for Hood and Jones [1987] model 1, which was therefore not used in this study. Consideration of a more complete projection scheme would eliminate this anomaly.

In addition to the simplified oxide and mineral analyses, Table 2 gives the whole rock value of $Mg' \equiv MgO/(MgO + FeO)$ by mole, and the calculated density at standard temperature and pressure. Densities of mineral species were based on values for magnesium and iron end-members tabulated in BVSP [1981], interpolated linearly for intermediate compositions.

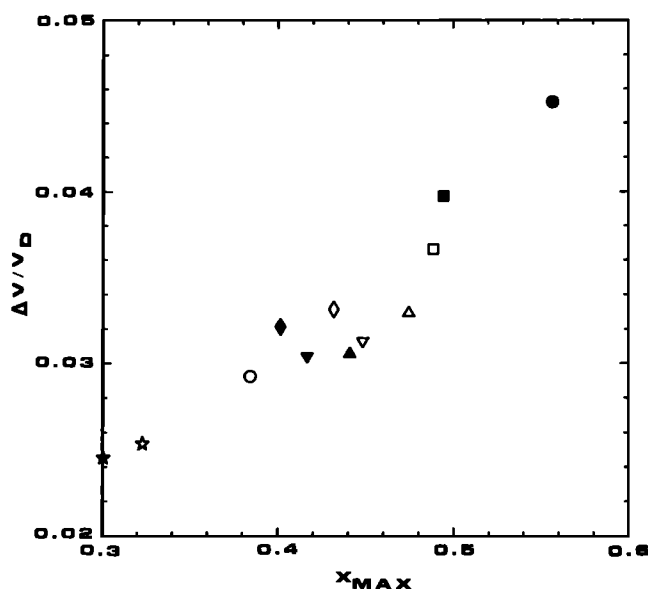


Fig. 1. Volume change on differentiation versus maximum mass of basalt extractable from unit mass of primordial Moon. Symbol shape indicates bulk Moon model: circle, M1; square, M2; triangle, M3; inverted triangle, M4; diamond, M5; star, M6. Symbol shading indicates basalt model: solid, B1; open, B2. See Table 1 for sources of models.

For each combination of bulk Moon and basalt composition, we next calculate the maximum amount of the latter which can be extracted from the former. The yield is usually limited by the exhaustion of one of the constituent oxides in the residue. In some cases, however, a composition corresponding to the vanishing of one of the normative minerals is achieved first. Included in Table 2 are the compositions of the limiting-case residues and of course their densities. (That the residues are generally less dense than the basalt is perhaps surprising, but does not preclude upward differentiation, since the basaltic melt will be buoyant, with $\rho \approx 3000 \text{ kg m}^{-3}$ [Bottinga and Weill, 1970].) The volume change $\Delta V/V_d$ for complete basalt extraction is given for each pair of compositions in Table 3, along with the maximum yield (as a ratio by mass) and the element or mineral whose abundance is limiting. The results are also summarized in Figure 1.

As expected, our estimates of the differentiation volume change vary but are all in the range of a few percent. It is clear from Figure 1 that the primary factor determining $\Delta V/V_d$ is the fraction X_{max} of the primordial rock which can be converted to basalt. Since the yield is in most cases limited by CaO, there is an accompanying trend toward larger values of $\Delta V/V_d$ for bulk Moon models which are richer in CaO and Al_2O_3 (our lower-numbered models). Other influences are more subtle, but one at least

TABLE 3. $\Delta V/V_d$, X_{max} , and Limiting Element/Mineral

	M1	M2	M3	M4	M5	M6
B1	0.0452	0.0396	0.0304	0.0305	0.0321	0.0245
	0.5567	0.4942	0.4408	0.4165	0.4013	0.3003
	Fe	Ca	Ca	Ca	Ca	Ca
B2	0.0292	0.0365	0.0328	0.0311	0.0331	0.0253
	0.3843	0.4885	0.4743	0.4481	0.4316	0.3230
	Ol	Fe	Ca	Ca	Ca	Ca

may be remarked: basalt B2, which is richer in FeO and MgO, poorer in SiO₂ and CaO than B1, has higher X_{max} and $\Delta V/V|_d$ with the calcium-aluminum-poor Moon models. When paired with M1 and M2, however, it is limited respectively by olivine (i.e., too much SiO₂ left in the residue) and iron, and lower values of $\Delta V/V|_d$ result. In light of these trends, it is interesting that Hood and Jones [1987] report the greatest success in fitting seismic velocity constraints for their most aluminous bulk models: our M2 and the "unadjusted" precursor of M1. This is in part a consequence of the increase of seismic velocities in the lower mantle calculated by Nakamura [1983], favoring the presence of garnet. The earlier velocity profiles of Goins [1978] showed a drop in velocity between 400- and 480-km depth, interpreted as a consequence of the presence of Fe-FeS [Buck and Toksöz, 1980; BVSP, 1981] or less likely, a decrease in Mg'. Values of $\Delta V/V|_d$ towards the upper end of our range would thus seem to be favored, unless the primary melt was especially iron-magnesium rich. We will nevertheless adopt the more conservative values $\Delta V/V|_d = 0.03$, $X_{max} = 0.4$ in our exploration of the importance of differentiation in the remainder of this paper.

COMPETITION BETWEEN THERMAL AND COMPOSITIONAL EFFECTS

The net expansion resulting from differentiation of lunar material is so large that it is likely to be an important factor in the volume history of the Moon unless formation involved either complete melting or such low temperatures that the solidus was never thereafter achieved. The typical 3% expansion arrived at above corresponds, for example, to the effect of heating olivine from 425 to 1350 K [Skinner, 1962]. The upper limits on the contributions to expansion of the lunar interior from warming and from basalt formation are thus nearly equal. In this section we will attempt to estimate the minimum amount of differentiation required to offset the expected thermal contraction of the outer layers of the Moon. In this way we may place a rough limit on the "hottest" initial state consistent with a nearly constant lunar volume, subject to the minimum number of questionable assumptions.

The thermal contraction of a hot Moon is largely independent of the details of heat transport in the interior, since it is dominated by the thickening of a cold boundary layer near the surface. Detailed models of the lunar magma ocean [e.g., Solomon and Longhi, 1977] indicate that the outer regions of the Moon will have largely or entirely frozen by 3.8 Gyr ago. The boundary layer may therefore be modeled as a uniform, conductively cooling solid. We adopt a thermal diffusivity $\kappa = 7.5 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$ [Toksöz and Solomon, 1973] and a coefficient of thermal expansion $\alpha = 4 \times 10^{-5} \text{ K}^{-1}$ [Solomon and Chaikin, 1976] as representative for a range of temperatures. In 3.8 Gyr the thermal boundary layer will have achieved a thickness of the order of $\sqrt{\kappa t} \simeq 300 \text{ km}$. This is sufficiently thin compared with the radius of the Moon that we can ignore the effects of spherical geometry to a reasonable first approximation. (In fact, the effects of our simple assumptions about the boundary layer geometry and the temperature-independent thermal expansion coefficient largely cancel, resulting in a fortuitously good agreement with the results of the more detailed model presented in the next section.) Convection, if present, does not greatly mod-

ify this estimate [Cassen et al., 1979; Schubert et al., 1980]. Across this layer the temperature varies from the solidus, $\sim 1350 \text{ K}$, to the equilibrium surface temperature, $\sim 250 \text{ K}$ [Toksöz and Solomon, 1973], giving an average cooling of $\sim 550 \text{ K}$. The resulting change in the radius of the Moon is roughly $\sim 5.5 \text{ km}$. This is comparable to the net contraction for the hottest thermal models of Solomon and Chaikin [1976], which do not, of course, include differentiation.

In order to yield a net radius change of less than 1 km, this thermal contraction must be offset by 4.5 km of expansion due to differentiation (we ignore warming of the interior). Using $\Delta V/V|_d \simeq 0.03$, we see that $\sim 26\%$ of the volume of the Moon (equivalent to a sphere 1110 km in diameter) must be completely differentiated. If α is allowed to be as low as $2 \times 10^{-5} \text{ K}^{-1}$, this value decreases to 810 km (10% of the Moon's volume). Allowing the core to start out cold will further decrease its required size to as little as 680 km, a fact of interest if one seeks the Moon model with the greatest degree of initial melting, rather than the most uniformly high temperature. Hypothetically, then, the radius constraint may be satisfied while permitting by far the majority of the Moon's mass to be molten at the time of formation.

How much basaltic melt is produced by the differentiation of a 1110-km lunar core? According to the results of section 2, a total of $\sim 0.4 \text{ m}^3$ of basalt may be extracted from 1 m^3 of primordial lunar material. Approximately 10% of the lunar volume of melt is thus produced, most of which must refreeze before reaching the surface. The total volume of mare basalt erupted onto the surface has been estimated at $\sim 1\%$ of the volume of the crust, or $\sim 0.1\%$ of the lunar volume [Head, 1975]. The ratio of melt production to extrusion is thus of the order of 100 : 1. The complexity and variety of differentiation patterns in the observed lunar basalts argue for remelting and reworking of the primary gabbro, so that the ratio of intrusive to extrusive activity is probably somewhat greater. Though large, this ratio cannot be excluded on the basis of any physical arguments (see section 5).

Disposition of 10% of the lunar volume of gabbroic material in the outer regions of the Moon is not necessarily inconsistent with the observed seismic velocity profile [Nakamura, 1983]. We have calculated approximate seismic velocities for our model basalts in the plagioclase and spinel stability fields, based on the mineral elastic properties tabulated by Hood and Jones [1987], our whole rock velocities being volume averages of those for the individual minerals. We concentrate on the shear wave velocity V_s , on which the observational constraints are stronger. In the spinel field, V_s ranges from $\sim 4.27 \text{ km s}^{-1}$ at 200-km depth to $\sim 4.15 \text{ km s}^{-1}$ at 500 km. Nakamura's [1983] estimate $V_s \simeq 4.25 \pm 0.1 \text{ km s}^{-1}$ is thus consistent even with a pure gabbro layer at this depth. In the plagioclase field, Nakamura's velocity is higher ($4.49 \pm 0.03 \text{ km s}^{-1}$) and that for the gabbro is lower ($4.23\text{--}4.06 \text{ km s}^{-1}$). Models of this region as the source from which the crust was derived typically have V_s in excess of observation, however, at least near the surface [Hood and Jones, 1987]. The data thus admit of a substantial admixture of gabbro into the magma-ocean residue, more than sufficient to accommodate the amount of melt envisioned here. These arguments are far from conclusive, but seismic velocity measurements do not seem to rule out the emplacement of a substantial quantity of gabbro above 500-km depth in the Moon.

IMPLICATIONS FOR LUNAR HISTORY

The calculations of the previous section indicate that differentiation of a small fraction of the lunar interior could offset substantial thermal contraction. They do not guarantee that a balance leading to radius variations of less than ± 1 km in the last 3.8 Gyr can actually be attained. Such a balance would depend on the timing of differentiation and thermal contraction and at best would likely be imperfect, satisfying the radius constraint but resulting in more than the minimum amount of differentiation over time. (For example, any basalt formation before 3.8 Gyr ago is "wasted.") Unfortunately, the initial conditions (if any) which will lead to an acceptable radius history can only be determined in the context of specific assumptions about the thermal evolution of the Moon. Our purpose in this section is not to formulate the definitive model of the Moon's thermal and differentiation history. Rather, we are content to show that the incorporation of $\Delta V/V|_d$ in a simple thermal model results in acceptable radius variations for substantially "hotter" initial states than would otherwise be possible.

We have chosen to consider a conductive thermal evolution model, similar in many respects to that described by *Töksöz and Solomon* [1973], and employed also in *Solomon and Chaikin* [1976] and *Solomon* [1977]. It might seem strange to pursue a conductive model, in view of the almost universal acceptance of subsolidus convection as the primary means of eliminating heat from homogeneous terrestrial bodies, even those as small as the Moon [cf. *Tozer*, 1972; *Schubert et al.*, 1977]. We have chosen this approach for the following reasons. (1) We wish to compare our results with the only previous well-documented models of radius change, and those models are conductive. (2) Plausible models for the early Moon may involve a prolonged conductive phase for the deep interior if the core is colder than the upper layers. (3) There is no generally accepted scheme or recipe for modeling convection coupled to differentiation. (4) We dispute the conventional wisdom that subsolidus convection is an inevitable eventual state for the deep interior of the Moon, in light of the possibility that substantial com-

positional gradients may arise during irreversible differentiation. (5) Some of the results may be insensitive to the details of the heat transport mechanism (a point discussed further at the end of this section).

We employ the same explicit finite difference scheme as *Solomon and coworkers* to solve the thermal conduction equation in spherical coordinates,

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k(r, T) \frac{\partial T}{\partial r} \right) + H(r, t) \quad (3)$$

where ρ is density, C_p is specific heat, and H is the contribution of heat sources. We adopt a number of the same values of physical parameters (Table 4) as the above authors. We also characterize our initial state by the same quantities: a central temperature T_c and a depth Z_0 . Above Z_0 the initial temperature follows the solidus, while below it varies quadratically with depth between the solidus and T_c . Our model uses a simplified basalt solidus, however, which varies quadratically with depth (linearly with pressure) from 1348 K at the surface to 1883 K at the center of the Moon [*Solomon and Töksöz*, 1973]. In addition, we adopt the lower crustal thermal conductivity of *Binder and Lange* [1980].

Where our model necessarily departs from *Töksöz and Solomon* [1973] is in the treatment of melting. Only the originally undifferentiated region below Z_0 is permitted to melt. (The outer layers, once solidified from the magma ocean, never become hot enough for remelting to occur.) The quantity of melt X previously extracted from the material at each depth is maintained as part of the calculation; it is initially zero. Generally, $\partial T / \partial t$ is determined from (3), and $\partial X / \partial t = 0$. If, however, the temperature is at the solidus, the righthand side of (3) is positive, and X is less than $X_{max} = 0.4$ (section 2), then $\partial T / \partial t = 0$, and melting occurs according to

$$\rho L \frac{\partial X}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k(r, T) \frac{\partial T}{\partial r} \right) + H(r, t) \quad (4)$$

We adopt a latent heat of fusion $L = 4 \times 10^5 \text{ J kg}^{-1}$ [*Binder*

TABLE 4. Parameters for Thermal Evolution Models

Parameter	Value
ρ , kg m^{-3}	3340 ^a
C_p , $\text{J kg}^{-1} \text{K}^{-1}$	1200 ^a
L , $\text{J kg}^{-1} \text{K}^{-1}$	4×10^5 ^a
T_{sol} , K	$1348 + 0.113P^b$
k , $\text{W m}^{-1} \text{K}^{-1}$	$\frac{418.4}{30.6 + 0.21T} + \max[0, 2.3 \times 10^{-3}(T - 500)]^{c,d}$
α , K^{-1}	$\frac{2.2 \times 10^{-5} - 1.05T^{-2} + 1.44 \times 10^{-8}T}{1 + 2.2 \times 10^{-5}T + 1.05T^{-1} + 7.2 \times 10^{-9}T^2}$ ^{c,e}
$\frac{\Delta V}{V} _d$	0.03
X_{max}	0.4
K/U	2000 ^a
Th/U	3.7 ^f
R , km	1740
Δr , km	20
Δt , Myr	2
$T(R)$, K	253 ^g

^a *Töksöz and Solomon* [1973].

^b P in MPa.

^c T in K.

^d *Schatz and Simmons* [1972] in mantle. Values 2.0, 0.5 from *Binder and Lange* [1980] used in crust (20–80 km) and regolith (0–20 km).

^e *Skinner* [1962] for olivine.

^f *Solomon* [1977].

^g *Langseth et al.* [1972].

and Lange, 1980]. The melt is assumed to migrate upward instantaneously and refreeze in the crust (depths less than 80 km), depositing its latent heat. This must be understood as purely a matter of convenience in “bookkeeping” for the energy budget; we do not mean to imply that the 80-km lunar crust was formed from the relatively late-forming melt that characterizes our model. Radiogenic heat sources are also assumed to partition entirely into the melt, such that by the time $X = X_{max}$, their concentration in the residue vanishes. Both latent heat and radionuclides are deposited in proportions varying exponentially with depth in the crust; a skin depth of 30 km is assumed but has little impact on the results. We do not include in our model the inward transport of heat and mass which is implied by the shrinkage of the residuum as melt is extracted. Nor do we attempt to describe the higher-temperature melting which would occur after basalt extraction is complete, since we find that $X = X_{max}$ is not reached in our models.

The radius history of the Moon is evaluated by integrating over time the expression

$$\frac{\partial \Delta R}{\partial t} = \frac{1}{R^2} \int_0^R \left(\alpha(T) \frac{\partial T}{\partial t} + \frac{\Delta V}{V} \Big|_d \frac{1}{X_{max}} \frac{\partial X}{\partial t} \right) r^2 dr \quad (5)$$

which includes both thermal and differentiation effects in an approximate way. The initial condition is chosen a posteriori such that $\Delta R = 0$ at the present day. It is sufficient to use the present-day value of the lunar radius R on the right-hand side of this equation.

We evaluate the success of our thermal histories against several constraints. The most important is that on lithospheric stress resulting from changes in radius. A brief comment on the relationship between stress and radius in a differentiating body is in order. The ± 1 km limit on radius change is derived from a limit of roughly ± 1 kbar on the tangential stress σ_t sustainable by the lithosphere without fracture, and the relationship

$$\sigma_t(R) = \frac{E}{1-\nu} \frac{\Delta R}{R} \quad (6)$$

where E is the Young's modulus and ν the Poisson ratio. This expression may be obtained from the general formula for σ_t versus depth in a uniform thermoelastic sphere [Solomon and Chaikin, 1976]. Solomon [1986] makes clear, however, that (6) is much more general, applying to the outermost layer of a planet, considered as an elastic shell, regardless of the properties of the interior. This generality is important because a uniform elastic sphere is a poor model for a differentiating Moon. The appropriate model is a relatively thin elastic shell surrounding an effectively fluid region, i.e., a balloon [Niven, 1972]. In the deep interior, the temperature is at or near the solidus, allowing relaxation of elastic stresses by creep. Nearer the surface, stresses are relieved by the opening of fractures and intrusion of magma. Only the outermost region can be treated as elastic, but this is sufficient for the usual ± 1 km limit on radius variation to obtain. The Moon differs from the usual kind of balloon in one important way, however. Its interior, though effectively fluid (zero shear modulus) is no more compressible than the exterior. The restraining effect of the elastic shell on the expansion of its contents may therefore be neglected to a good approximation, as was implicitly done in (5).

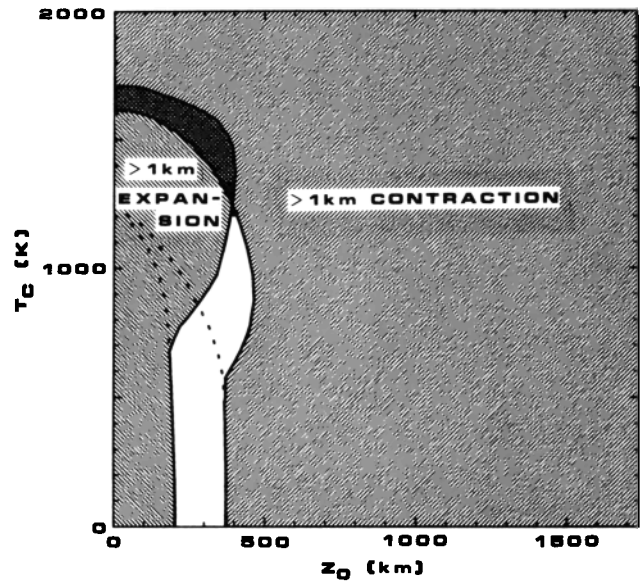


Fig. 2. Lunar thermal evolution models with 20 ppb bulk uranium content. Shaded regions of parameter space are in violation of the 1-km limit on expansion or contraction (or both) since 3.8 Gyr ago. Dotted contours indicate apparent states of ± 1 km radius change if the effect of differentiation is neglected. Dot indicates a model with central temperature $T_c = 1210$ K, and magma ocean depth $Z_0 = 400$ km, which appears in Figures 3–5.

In addition to applying the radius constraint, we compare the present-day surface heat flux to the recent reinterpretation of the Apollo 17 measurement by Warren and Rasmussen [1987]. They modeled the effects of lateral inhomogeneities on thermal conductivity as well as a strongly insulating megaregolith, and concluded that an appropriate globally averaged value of the heat flux is probably 12 mW m^{-2} . We vary the lunar bulk content of uranium (in addition to T_c and Z_0) to attempt to reproduce this value. Finally, we also keep track of the time of transition (if any) between net expansion and contraction of the Moon. Solomon and Head [1979] conjectured that this transition may be responsible for the end of rille and graben formation 3.6 Gyr ago.

Figure 2 summarizes the results of our model calculations for a bulk uranium content of 20 ppb. The limits on expansion and contraction are simultaneously satisfied for a range of models (unshaded region) with $T_c \lesssim 1210$ K and $210 \lesssim Z_0 \lesssim 470$ km. The maximum value of T_c is attained for $Z_0 \simeq 405$ km. Were one to neglect $\Delta V/V|_d$, in contrast, the maximum acceptable T_c would be nearly the same, but the magma ocean would disappear ($Z_0 \rightarrow 0$) as $T_c \rightarrow 1200$ K. Only for a much colder core would a 200–400 km magma ocean be acceptable. This is in keeping with the results of Solomon and Chaikin [1976], whose model resembles ours but with 30 ppb of uranium and a less insulating crust.

The present-day heat flux at the surface is 11.8 mW m^{-2} for our hottest acceptable model in Figure 2, falling off to $\sim 8 \text{ mW m}^{-2}$ for the coldest models shown. This is in agreement with Warren and Rasmussen [1987], who calculated the present-day flux with a very different treatment of the lunar interior and arrived at a uranium content of 20–21 ppb. If we seek to match Pullan and Lambek's [1980] temperature of 973 K at 240-km depth rather than the heat

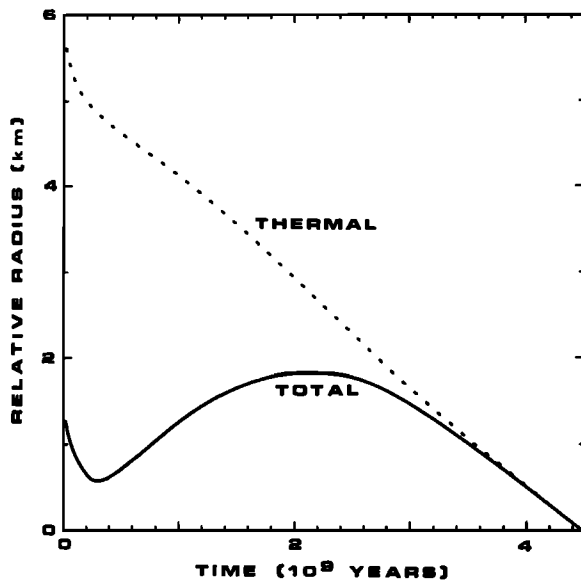


Fig. 3. Relative radius of the Moon versus time for the thermal evolution model indicated by a dot in Figure 2. Solid curve shows net radius change, including expansion due to differentiation, relative to present day. Dotted curve shows contribution due to thermal contraction.

flow, we can do so for 14 ppb uranium, $T_c = 1522$ K, and $Z_0 = 160$ km. This is the same uranium content for which Warren and Rasmussen [1987] satisfied the 240-km temperature constraint, and it is the hottest model with this uranium content that has an acceptable radius history.

We will examine more closely a single thermal history calculated with an uranium abundance of 20 ppb, $T_c = 1210$ K and $Z_0 = 400$ km, indicated by the dot in Fig. 2. Figure 3 (solid curve) shows the evolution of the lunar radius with

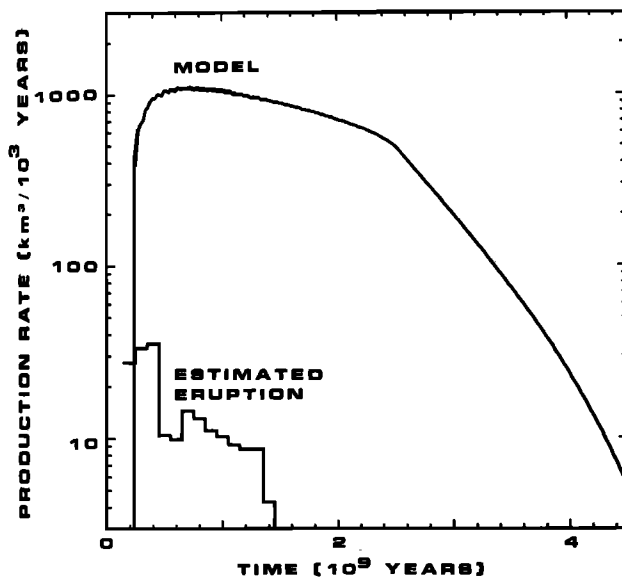


Fig. 4. Rates of basaltic melt production. Upper curve shows theoretical melting rate for the model indicated by dot in Figure 2. Break in slope at ~ 2 Gyr is due to the front at which melting begins reaching the center of the body. Lower curve shows estimate of volcanic eruption rate based on unit thicknesses and crater density ages [BVSP, 1981]. Overall ratio of melt formation to eruption is $\sim 98:1$.

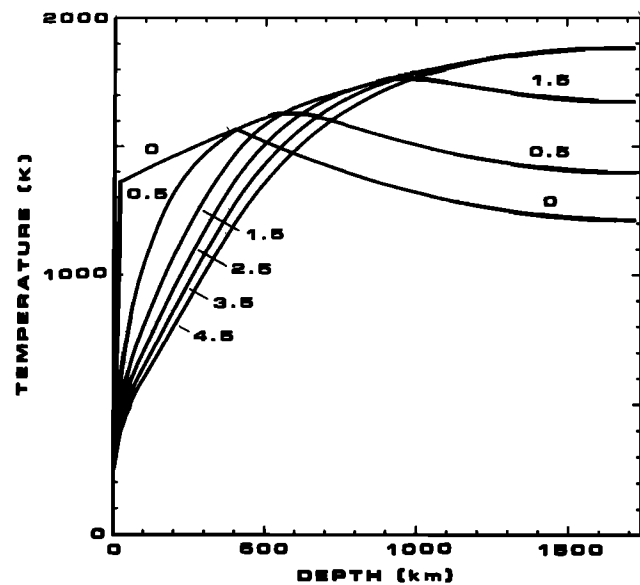


Fig. 5. Representative selenotherms for the model indicated by dot in Figure 2. Temperature versus depth is indicated for times of 0, 0.5, 1.5, 2.5, 3.5, and 4.5 Gyr after formation. Curves are bounded above by the assumed basalt solidus (quadratic in depth). Surface heat flux at present is 11.8 mW m^{-2} .

time. As expected for this near-limiting case, the Moon has been both ~ 1 km larger and ~ 1 km smaller since 3.8 Gyr ago than its size at that time. For purposes of comparison, the dotted curve shows the radius variation for the same thermal history but ignoring $\Delta V/V|_d$. A monotonic decrease of nearly 4.5 km in the last 3.8 Gyr is erroneously predicted. Note that the maximum lunar radius (including differentiation effects) occurred 2.3 Gyr ago. This is a general feature of our models: as Z_0 is decreased, there is an abrupt transition from monotonically decreasing radius histories to ones with a relatively recent maximum. To push the time of this maximum back to 3.6 Gyr ago requires a central temperature in excess of ~ 1500 K, which is not consistent with the constraint on the magnitude of ΔR .

The instantaneous rate of melt generation is shown in Figure 4, along with an estimate of the rates of eruption based on thicknesses and ages of major volcanic units [BVSP, 1981]. The ratio of melt production to effusion is as low as 30:1 in the early, "upland" flooding stage, and about 100:1 during the flooding of the maria. Roughly two thirds of the total melting predicted by the model occurs before the end of major mare volcanism ~ 3 Gyr ago (a fact disguised somewhat by the logarithmic scale of Figure 4). The rate of melting continues to decline slowly until about 2 Gyr ago when the onset-of-melting front reaches the center of the Moon, then falls off more rapidly. This is comparable to the ages estimated from crater densities for the youngest mare basalts [BVSP, 1981]. Overall about 98 times as much melt forms in our model as is believed to have been evolved onto the surface. This is remarkably similar to the rough estimate made in the previous section.

Finally, in Figure 5 we present selenotherms at several times during the evolution of our hottest model (solid curves). The assumed basalt solidus is indicated by the dashed curve, mostly hidden by the envelope of the selenotherms. Partial melting occurs at all depths greater

than Z_0 over the course of time. About 60% of the maximum possible amount of basalt is extracted from most parts of the core, dropping to zero at its top, where cooling is rapid. Because of the density difference between primordial mantle and residuum, this situation is gravitationally unstable, and the core will probably overturn [Hood, 1986] and achieve a uniform state of roughly 54% complete basalt extraction. The STP density of such a mixture of primordial and residual material is roughly $3350\text{--}3450\text{ kg m}^{-3}$ (depending on the compositional models chosen). It may be the case, as Taylor [1982] has objected, that basaltic material would be denser than is consistent with observations if it existed in the lower mantle of the Moon. The same is not true for the residuum from which basalt has been extracted. Our STP density is similar to or slightly less than typical lower-mantle densities derived from geophysical models [Hood and Jones, 1987].

Melt is still being formed below about 1300 km. This is consistent with (though not required by) the possible increase in seismic shear wave attenuation below 1100 km. Our present-day selenotherm is generally consistent with estimates of temperature in the lunar interior based on electrical conductivity and lithospheric rigidity. These thermal indicators are discussed at length by Hood [1986].

The thermal and volumetric evolution model presented in this section is far from definitive. Its primary assets are simplicity and similarity to models from which conclusions about the Moon's initial thermal state have been drawn in the past. Much refinement in the treatment of the differentiation process is possible, e.g., calculation of the pressure- and temperature-dependent volume change on a point-by-point basis, using a more realistic mineral scheme, and proper treatment of mass, energy, and radiogenic element transport by the melt phase. It is not likely, however, that these improvements would change the result substantially. The possible qualitative effect of subsolidus convection on the thermal and differentiation history is of greater interest at this point. Existing convective lunar thermal models [Tóksöz *et al.*, 1978; Cassen *et al.*, 1979] show that horizontally averaged temperatures can reach the solidus and mantle-wide partial melting can occur. Further melting will be possible in the upwelling regions even after the average temperature has dropped below the solidus. Whether the extent and timing of melting can be made consistent with a near-constant radius history remains to be seen. It is possible that subsolidus convection may actually help, in the sense that more efficient heat transport will increase the initial temperatures needed to achieve a given amount of melting. We reiterate our earlier point, however, that existing convection calculations cannot be adapted to our purpose because they omit the effects of irreversible differentiation, which are large in our case.

DISCUSSION

Discussion. Both our scaling estimates based on the magnitude of $\Delta V/V|_d$ and the particular thermal evolution model we have chosen to investigate suggest that the observed lack of lunar tectonism may be reconciled with an initial state in which $\gtrsim 50\%$ of the Moon is molten, and the rest within a few hundred Kelvin of the solidus temperature. This is just one of many possible outcomes; we have not chosen to meet the challenge of constructing a "definitive" thermal and compositional history of the Moon. Indeed, we doubt that this challenge can be met at present. However,

the simple estimates of the trade-off between compositional and thermal effects on volume are in general confirmed by detailed (albeit imperfect) models, in support of our view that a hot early Moon cannot be discounted. Two issues naturally arise from this conclusion: Why is the volume of mare basalts so small, and how can we reconcile our picture of a partially molten early Moon with the initial complete melting of lunar source material in a giant impact scenario?

We assert that our conclusion of $\sim 10:1$ intrusive (gabbro) to extrusive (basalt) igneous activity is not at all unreasonable. While it is true that the ratio of intrusive to extrusive igneous activity in continental regions of the Earth is only $\sim 10:1$ [Crisp, 1984], it is more pertinent that basalt is not buoyant or barely buoyant when ascending near-surface conduits on the Moon [Walker and Hays, 1977; Bottinga and Weill, 1970]. An analysis of melt migration along cracks reveals that the local density contrast is very important to the ability of melt to migrate upward [Stevenson, 1982; Spence *et al.*, 1987] (but see Crawford and Stevenson, [1987] for a potential counterexample), so the ability of basalt to reach the surface of the Moon is not aided by increasing the depth of the source region. One could legitimately adopt the view that it is remarkable that the Moon has any mare basalts! Conceivably, impact played a role in aiding their eruption.

The problem of lunar origin by giant impact requires more work [cf. Stevenson, 1987], but it is possible that this scenario is consistent with an initial Moon that is only partially molten and perhaps even subsolidus at depth. A. C. Thompson and D. J. Stevenson, (Gravitational instability of two-phase disks and the origin of the Moon, submitted to *Astrophysical Journal*, 1987) find that the impact-generated fluid disk may cool at the periphery, while generating a large number ($10\text{--}10^2$) of protomoons. These bodies eventually collide to form the Moon as we know it, but sufficient time elapses for each protomoon to grow a chill crust. If these crusts rupture and founder, then a substantial fraction (potentially more than half) of the molten source material can freeze before accumulation. We can place an upper bound on the freezing as follows: Suppose there are N protomoons, each radiating at a temperature T_e for a time τ . The fraction x of material that freezes is then given by

$$\begin{aligned} xLM &= 4\pi r_p^2 \sigma T_e^4 \tau \cdot N \\ &= 4\pi R^2 \sigma T_e^4 \tau \cdot N^{\frac{1}{3}} \end{aligned} \quad (7)$$

where L is the latent heat of freezing, M is the mass of the Moon, R is the lunar radius, $r_p \simeq N^{-1/3}R$ is the protomoon radius, and σ is the Stefan-Boltzmann constant. We find

$$x \simeq 7 \left(\frac{T_e}{10^3 \text{ K}} \right)^4 \left(\frac{\tau}{10^3 \text{ yr}} \right) \quad (8)$$

Since an accumulation time $\tau \simeq 10^3$ yr or more is likely, x can be substantial even if T_e is reduced because only a fraction of the surface of a protomoon is molten at any instant. The newly formed Moon may consist of a central "core" of dense chill crust, surrounded by a magma ocean. These ideas are speculative but merit more work.

We conclude by remarking that even with the possibility that differentiation effects may partially cancel thermal contraction, it is astonishing that the Moon exhibits so little evidence for volume change. One should be wary of explanations that rely on fortuitous cancellation (a criticism that applies equally to the old thermal models and to our model).

Perhaps there is more to this puzzle than anyone has yet discerned.

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